# Tracking an Accelerated Target with a Nonlinear Constant Heading Model

**Rong Yang, Gee Wah Ng**
DSO National Laboratories
20 Science Park Drive
Singapore 118230
yrong@dso.org.sg
ngeewah@dso.org.sg

**Boon Poh Ng**
School of EEE
Nanyang Technological University
Blk S2, B2a-05, Nanyang Avenue
Singapore 639798
ebpng@ntu.edu.sg

**Abstract** – *This paper proposes a nonlinear model to track a maneuvering target in acceleration motion. The acceleration motion is traditionally modeled as a linear function with the state which consists of target position, speed and acceleration in the x, y and possibly z coordinates. The state elements in different coordinate are assumed uncoupled. However, This assumption is not generally true, as the state elements in different coordinates are correlated by the common target heading. Thus, a nonlinear constant heading model is suggested in this paper. To implement this nonlinear model, a two-stage least squares method is developed for track initiation, and an extended Kalman filter (EKF) and an unscented Kalman filter (UKF) are proposed to estimate the state in track maintenance. Performance of the nonlinear model is demonstrated through simulation data, and results show that the proposed nonlinear model outperforms the traditional linear model.*

**Keywords:** Maneuvering target, acceleration model, nonlinear tracking, track initiation

## 1 Introduction

The correctly defined models, which describe various target motion patterns, are the fundamental for the tracking system. The main motion patterns for a vehicle include constant velocity motion, constant acceleration motion and turn motion. In this paper, we focus on the mathematical model of constant acceleration motion. A natural acceleration motion is of that a constant force acting on a target at the target heading direction resulting that the target moves in a straight line with constantly changing speed. The constraint on such motion in 2D space is

$$\frac{\mathbf{a}}{|\mathbf{a}|} = \frac{\mathbf{s}}{|\mathbf{s}|} \tag{1}$$

or

$$\frac{\dot{x}}{\dot{y}} = \frac{\ddot{x}}{\ddot{y}} \tag{2}$$

where $\mathbf{a}$ and $\mathbf{s}$ are the target acceleration vector and velocity vector respectively, $\dot{x}, \dot{y}, \ddot{x}, \ddot{y}$ are the target speed and acceleration in x and y coordinates respectively. However, the commonly used existing acceleration models are all based on the assumption that the coupling among the coordinates can be neglected for simplicity, and the state elements on each coordinate are independent. This assumption does not guarantee the target moves in a straight line. Thus a model error is introduced and leads to a decrease in track accuracy. The aim of this paper is to design an easily implemented model to describe the acceleration motion without such model error.

The acceleration motion pattern has been modeled in many forms over the past four decades [1][2][3]. A popular acceleration maneuvering model is designed by Singer [4]. The target acceleration is modeled as a first-order Markov process with the state $[\begin{array}{ccc} x & \dot{x} & \ddot{x} \end{array}]^T$, which represents the target position, speed and acceleration in x coordinate. The Singer's model needs a priori knowledge on target maneuver time $\tau$ and the model of acceleration probability density to determine its transition equation. This model is a mixture of acceleration model and constant velocity model rather than a dedicated acceleration model. The acceleration can also be simply modeled as white Gaussian noise in the constant velocity (CV) model [2]. The state vector is $[\begin{array}{cc} x & \dot{x} \end{array}]^T$, and the acceleration $\ddot{x}$ is totally merged into the process noise in the CV model. It is equivalent to Singer's model when $\tau$ is sufficiently small. More precisely, the acceleration can be modeled by the constant acceleration (CA) model, which includes $\ddot{x}$ in the state vector as $[\begin{array}{ccc} x & \dot{x} & \ddot{x} \end{array}]^T$ [2]. The most popular CA model is referred to as the Wiener process acceleration model. The process noise in this model is the maximum acceleration increment expected over the scan interval $T$. Another version of CA model is called the white jerk model. The white jerk model sets the process noise as white Gaussian on the expected target jerk (i.e. acceleration derivative). It can also be viewed as a special case of Singer model when $\tau$ is much greater than scan interval.

All the models mentioned above are linear models, and the states are only represented in one coordinate. For multiple coordinates, the states are simply augmented, eg. the state becomes $[\ x \ \ \dot{x} \ \ \ddot{x} \ \ y \ \ \dot{y} \ \ \ddot{y}\ ]^T$ in the 2D CA model. Although the augmentation makes the models being implemented easily, it is too coarse that the correlation among coordinates is totally ignored and leads to model error.

In this paper, we propose a nonlinear constant heading model to track the constant acceleration target. The nonlinear model is implemented by an extended Kalman filter (EKF) and an unscented Kalman filter (UKF), and a two-stage least squares method is developed such that the target initial state and its error covariance can be efficiently estimated.

The rest of this paper is organized as follows. Section 2 proves the error caused by the uncoupled linear acceleration models, and the nonlinear constant heading model is proposed to overcome this model error. Section 3 suggests a two-stage least squares method for track initiation. Section 4 shows the simulation results, which present the performance comparisons on the uncoupled linear model and the proposed nonlinear model. Cramer-Rao lower bound (CRLB) is also demonstrated in the results. Finally, the conclusion is drawn in Section 5.

# 2 Acceleration models

In this section, we firstly present the model error generated by the uncoupled acceleration model. A nonlinear acceleration model to overcome this model error is then proposed. The nonlinear estimation algorithms for the proposed model are also discussed.

## 2.1 Model error of the uncoupled acceleration model

The state vector $\mathbf{x}$ of a generic uncoupled acceleration model in 2D space is defined as

$$\mathbf{x} = [\ x \ \ \dot{x} \ \ \ddot{x} \ \ y \ \ \dot{y} \ \ \ddot{y}\ ]^T \tag{3}$$

where $x$ and $y$ are the target positions, $\dot{x}$ and $\dot{y}$ are the target speeds, $\ddot{x}$ and $\ddot{y}$ are the target accelerations in x and y coordinates, respectively. The transition and measurement equations are

$$\mathbf{x}(k+1) = \mathbf{F}_1\mathbf{x}(k) + \mathbf{w}_1(k) \tag{4}$$
$$\mathbf{z}(k+1) = \mathbf{H}_1\mathbf{x}(k+1) + \mathbf{v}(k+1) \tag{5}$$

where

$$\mathbf{F}_1 = \begin{bmatrix} 1 & T & 0.5T^2 & 0 & 0 & 0 \\ 0 & 1 & T & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & 0.5T^2 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$\mathbf{H}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{7}$$

$T$ is the scan interval. $k$ is the scan index, $\mathbf{w}_1$ is the process noise, which is assumed to be the white Gaussian with covariance $\mathbf{Q}_1$, to compensate the model error. $\mathbf{v}$ is the white Gaussian measurement noise with covariance $\mathbf{R}$.

The state $\mathbf{x}$ and its error covariance $\mathbf{P}$ are estimated by Kalman filter, and they are

$$\hat{\mathbf{x}}(k+1) = \mathbf{F}_1\hat{\mathbf{x}}(k) + \mathbf{K}(k+1)\tilde{\mathbf{z}}(k+1) \tag{8}$$
$$\mathbf{P}(k+1) = [\mathbf{I} - \mathbf{K}(k+1)\mathbf{H}_1]\mathbf{P}(k+1|k) \tag{9}$$

where $\mathbf{K}(k+1)$ is the Kalman gain, and

$$\tilde{\mathbf{z}}(k+1) = \mathbf{z}(k+1) - \mathbf{H}_1\mathbf{F}_1\hat{\mathbf{x}}(k) \tag{10}$$
$$\mathbf{P}(k+1|k) = \mathbf{F}_1\mathbf{P}(k)\mathbf{F}_1^T + \mathbf{Q}_1(k) \tag{11}$$

The estimated state in (8) is the optimal solution without constraint (2). Now we take the constraint (2) into the consideration. The constraint can be rewritten as

$$\mathbf{D}\mathbf{x}(k+1) = \mathbf{0}_{2\times1} \tag{12}$$

where $\mathbf{0}_{2\times1}$ is $[\ 0 \ \ 0\ ]^T$, and

$$\mathbf{D} = \begin{bmatrix} 0 & -1 & 0 & 0 & \tan(h) & 0 \\ 0 & 0 & -1 & 0 & 0 & \tan(h) \end{bmatrix} \tag{13}$$

$h$ is the target heading, and it is defined as a clockwise angle from the true north. The optimal state $\tilde{\mathbf{x}}(k+1)$ with constraint is

$$\tilde{\mathbf{x}}(k+1) = \hat{\mathbf{x}}(k+1) - \mathbf{x}_e \tag{14}$$

where $\mathbf{x}_e$ can be considered as the model error of the uncoupled CA model, and it can be computed from the maximum probability using the Lagrangian method. The result is

$$\mathbf{x}_e = \mathbf{P}(k+1)\mathbf{D}^T(\mathbf{D}\mathbf{P}(k+1)\mathbf{D}^T)^{-1}\mathbf{D}\hat{\mathbf{x}}(k+1) \tag{15}$$

The detailed steps can be found in [5].

## 2.2 Nonlinear constant heading acceleration model

One approach to solve the estimation with constraint is to reduce the state size. Obviously, the six elements in the 2D uncoupled acceleration model can be reduced to five elements with the constraint (2). Thus we suggest a constant heading acceleration model in this subsection. The state vector $\mathbf{x}$ and the measurement vector $\mathbf{z}$ are defined as

$$\mathbf{x} = [\ x \ \ y \ \ s \ \ a \ \ h\ ]^T \tag{16}$$
$$\mathbf{z} = [\ x_m \ \ y_m\ ]^T \tag{17}$$

where $x$ and $y$ are the target positions in x and y coordinates respectively, $s$, $a$ and $h$ represent the target speed, acceleration and heading respectively. $x_m$ and $y_m$ are the measured target positions in x and y coordinates respectively. The transition equation is nonlinear, and is described by

$$\mathbf{x}(k+1) = f(\mathbf{x}(k)) + \mathbf{w}(k) \tag{18}$$

where $\mathbf{w}$ is the white Gaussian process noise with covariance $\mathbf{Q}$, and

$$f = [\ f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5\ ]^T \tag{19}$$

$$\begin{aligned}
f_1 &= x(k) + [s(k)T + 0.5a(k)T^2]\sin h(k) && (20)\\
f_2 &= y(k) + [s(k)T + 0.5a(k)T^2]\cos h(k) && (21)\\
f_3 &= s(k) + a(k)T && (22)\\
f_4 &= a(k) && (23)\\
f_5 &= h(k) && (24)
\end{aligned}$$

The measurement equation is linear, and is defined as

$$\mathbf{z}(k+1) = H\mathbf{x}(k+1) + \mathbf{v}(k+1) \tag{25}$$

where

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

and $\mathbf{v}$ is the white Gaussian measurement noise with covariance $R$, which is assumed to be obtained from a priori knowledge.

The Jacobian matrix of $f(\mathbf{x})$ is defined as

$$\mathbf{F}_2(k) = \frac{\partial f(\mathbf{x}(k))}{\partial \mathbf{x}(k)} \tag{27}$$

Substituting (20)-(24) into (27), $F_2(k)$ is obtained as

$$\mathbf{F}_2(k) = \begin{bmatrix} 1 & 0 & F_{13}(k) & F_{14}(k) & F_{15}(k) \\ 0 & 1 & F_{23}(k) & F_{24}(k) & F_{25}(k) \\ 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{28}$$

where

$$\begin{aligned}
F_{13}(k) &= T\sin h(k) && (29)\\
F_{14}(k) &= 0.5T^2\sin h(k) && (30)\\
F_{15}(k) &= [s(k)T + 0.5a(k)T^2]\cos h(k) && (31)\\
F_{23}(k) &= T\cos h(k) && (32)\\
F_{24}(k) &= 0.5T^2\cos h(k) && (33)\\
F_{25}(k) &= -[s(k)T + 0.5a(k)T^2]\sin h(k) && (34)
\end{aligned}$$

It is clear that the proposed nonlinear model describes the acceleration motion more accurately than the uncoupled acceleration model. However, there is no optimal algorithm for nonlinear tracking. The suboptimal algorithms provide approximate solution, and performance will depend on the accuracy of the initial state and subsequent recursive nonlinear estimation.

Many suboptimum nonlinear estimation algorithms have been proposed in literature for approximating the pdf of nonlinear transformations. The most popular one is the EKF [6] [2]. It linearizes the nonlinear function through a Jacobian matrix, so that the state covariance can be transited linearly. If model nonlinearity is not high, the tracking result of an EKF is acceptable. A generally more accurate nonlinear tracking algorithm is the UKF. It is a sampling method suggested by Julier [7] [8]. A few sigma points are used to represent the pdf of a state, and these sigma points are propagated through the nonlinear functions. The mean and the covariance of the posterior state are then obtained through the transited sigma points. Since Jacobian matrix is no more required, the UKF can be implemented easily, and the UKF has generally shown better performance than the EKF [9]. Another attractive nonlinear tracking algorithm is the particle filtering (PF) introduced by Gordon [10]. It is a sequential Monte-Carlo method. Instead of a few sigma points in UKF, plenty of points are used to represent the state pdf. The particle filter has been applied to various nonlinear non-Gaussian tracking applications [11].

In this paper, we implement the proposed model by the EKF and the UKF. $\mathbf{F}_2$ in (27) is the Jacobian matrix used in the EKF. The PF is not considered as it is computational intensive.

# 3 Two-stage least squares method for track initiation

Track initiation is an important step in tracking system, especially for nonlinear tracking. A wrong initial value can mislead the state converging into a local optimum instead of the global optimum. A typical method is to view the problem as a nonlinear optimization problem which is formulated by the maximum likelihood (ML). Normally a batch process is applied to the first few scans of measurement. The nonlinear optimization problem can be solved by the Newton's algorithm iteratively [12] [13]. Another approach is the Markov chain Monte Carlo (MCMC) method [14], which suggests a sampling method based on the likelihood linked to the observation equation. Both methods compute the initial state through iteration until some predetermined conditions are met.

In order to initiate an accurate nonlinear state rapidly, we develop a two-stage least squares method without iteration process. The target initial state and its error covariance can be efficiently estimated through this method.

Given the measurements in the first $n$ scans

$$\mathbf{Z} = [\ \mathbf{z}(1) \quad \mathbf{z}(2) \quad \ldots \quad \mathbf{z}(n)\ ] \tag{35}$$

where $\mathbf{z}(i)$ is defined in (17), $i \in [1, n]$. The parameters, which are going to be estimated, are separated into two sets, $S_1$ and $S_2$. $S_1$ consists of information in the first coordinate, for example, the x coordinate. $S_2$ contains of information in another coordinate and the correlation between two coordinates which is related to the target heading. The $S_1$ and $S_2$ are defined as

$$S_1 = [\ x_0 \quad \dot{x}_0 \quad \ddot{x}\ ]^T \tag{36}$$

$$S_2 = [\ y_0 \quad r\ ]^T \tag{37}$$

where $x_0$, $\dot{x}_0$ and $\ddot{x}$ are the target initial position, initial speed and acceleration in x coordinate respectively. $y_0$ is the target initial position in y coordinate. r is the cotangent of the target heading, which is defined as

$$r = \cot h = \frac{\dot{y_0}}{\dot{x_0}} = \frac{\ddot{y}}{\ddot{x}} \tag{38}$$

These two sets are estimated in two stages, which are described as follows.

Stage one estimates $S_1$. The problem is formulated as

$$\mathbf{Z}(1,:)^T = AS_1 + \mathbf{e}_x \tag{39}$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & T & 0.5T^2 \\ 1 & 2T & 0.5(2T)^2 \\ \vdots & \vdots & \vdots \\ 1 & (n-1)T & 0.5[(n-1)T]^2 \end{bmatrix} \tag{40}$$

$$\mathbf{e}_x = [\ e_x(1) \quad \cdots \quad e_x(n)\ ]^T \tag{41}$$

$e_x(i)$ is the measurement error in x coordinate, $i \in [1, n]$, and the variance is $\sigma^2$.

Using the least squares method, the $S_1$ is estimated by

$$\hat{S}_1 = (A^T A)^{-1} A^T \mathbf{Z}(1,:)^T \tag{42}$$

$S_2$ is estimated in the second stage. By making use of the result obtained in the first stage, the measurement equation in the second coordinate can be written as

$$\mathbf{Z}(2,:)^T = BS_2 + \mathbf{e}_y \tag{43}$$

where

$$B = \begin{bmatrix} 1 & 0 \\ 1 & T\dot{x}_0 + 0.5T^2\ddot{x} \\ 1 & 2T\dot{x}_0 + 0.5(2T)^2\ddot{x} \\ \vdots & \vdots \\ 1 & (n-1)T\dot{x}_0 + 0.5[(n-1)T]^2\ddot{x} \end{bmatrix} \tag{44}$$

$$\mathbf{e}_y = [\ e_y(1) \quad \cdots \quad e_y(n)\ ]^T \tag{45}$$

$e_y(i)$ is the measurement error in y coordinate, $i \in [1, n]$, and the variance is $\sigma^2$.

The $S_2$ is estimated as

$$\hat{S}_2 = (B^T B)^{-1} B^T \mathbf{Z}(2,:)^T \tag{46}$$

The initial states defined in (16) for the first $n$ scans can then be computed from $\hat{S}_1$ and $\hat{S}_2$. The target positions are

$$x(1:n) = A\hat{S}_1 \tag{47}$$

$$y(1:n) = B\hat{S}_2 \tag{48}$$

The target speed, acceleration and heading are computed by

$$s(1:n) = \sqrt{1+r^2}(\dot{x} + U\ddot{x}) \tag{49}$$

$$a(1:n) = L\sqrt{1+r^2}\ddot{x} \tag{50}$$

$$h(1:n) = L\tan^{-1}(1/r) \tag{51}$$

where $L$ and $U$ are

$$U = \begin{bmatrix} 0 & 1 & \cdots & n-1 \end{bmatrix}_{1\times n} \tag{52}$$

$$L = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}_{1\times n} \tag{53}$$

The state error covariance $\mathbf{P}$ is difficult to be estimated analytically. This is because the error has been introduced to matrix $B$ by $\dot{x}_0$ and $\ddot{x}$ in the second stage estimation. It results in the error of $S_2$ is difficult to be estimated. Thus, except the position $x$, the error statistics for other state elements have to be obtained through approximation and Monte Carlo test. In this paper, we compute and approximate the error variances for the first four state elements, namely, $x$, $y$, $s$ and $a$ as

$$\sigma_x^2 = \Sigma(1,1) \tag{54}$$

$$\sigma_y^2 \approx \Sigma(1,1) \tag{55}$$

$$\sigma_s^2 \approx \alpha\Sigma(2,2) \tag{56}$$

$$\sigma_a^2 \approx \alpha\Sigma(3,3) \tag{57}$$

where

$$\Sigma = (A^T A)^{-1}\sigma^2 \tag{58}$$

$\alpha$ is a scalar. The standard deviation on heading error $\sigma_h$ has to be obtained through a Monte Carlo experiment. We conducted the experiment on different measurement errors $\sigma$ and different number of initial scans. Figure 1 shows the result obtained from 500 Monte Carlo runs. It is seen that $\sigma_h$ is reduced when the number of initial scans increased and the measurement error $\sigma$ reduced.

The state error covariance $\mathbf{P}(n)$ at the $n$th scan is approximated by a diagonal matrix as

$$P(n) = diag[\ \sigma_x^2 \quad \sigma_y^2 \quad \sigma_s^2 \quad \sigma_a^2 \quad \sigma_h^2\ ] \tag{59}$$

Now the initial states for the first $n$ scans and the state error covariance at scan $n$ are totally obtained. Since there is no iteration process, the computational cost is much less compared to the existing ML and MCMC methods.

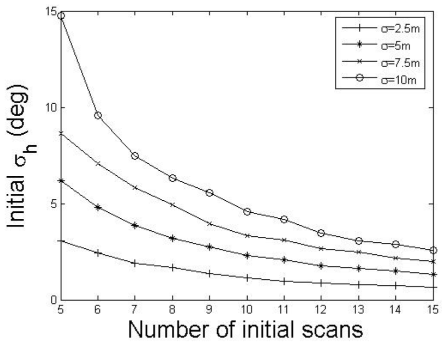


Figure 1: Target heading errors on different noise levels and different initial conditions

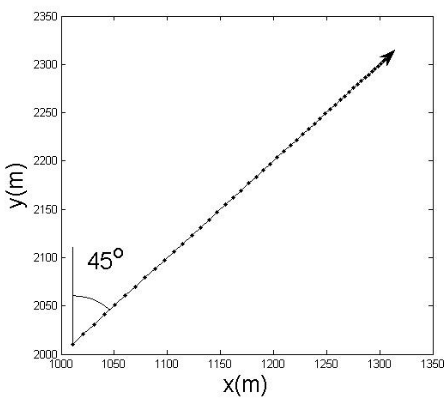


Figure 2: A test scenario to simulate a surface target taking a constant deceleration maneuvering from a cruising speed 15m/s to a slow speed 2.5m/s in 50sec

# 4 Simulation and analysis

This section demonstrates the performance of the proposed nonlinear acceleration model through simulation experiments. To illustrate the performance gained from the proposed nonlinear model, comparisons on tracking accuracy is made between the nonlinear model and the uncoupled acceleration model. The nonlinear model is implemented by the EKF and the UKF, and the nonlinear tracks are initiated by the two-stage least squares method. The Wiener CA model is selected as a representative of the uncoupled models, and the Kalman filter (KF) is used to estimate the state in this model. In the experiments, $\alpha$ in (56) and (57) is set to 2.

The test scenario is presented in Figure 2. It simulates a surface target taking a constant deceleration maneuvering from a cruising speed 15m/s to a slow speed 2.5m/s in 50sec. The target heading is 45 degrees. The target positions in x and y coordinates are detected every second. The measurement error is assumed to be white Gaussian in both x and y coordinates.

The experiments are conducted on different measurement errors and different initial conditions. The Root-Mean-Squared Error (RMSE) on target position is recorded for comparison. The position error at scan $k$ is computed by

$$p_{err}(k) = \sqrt{x_{err}^2(k) + y_{err}^2(k)} \tag{60}$$

where $x_{err}$ and $y_{err}$ are the differences between the estimated position and the ground truth in x and y coordinates respectively. The CRLB on position is also computed as a benchmark for performance evaluation. The CRLB can be obtained from the filtering information matrix recursively [15] [11] [16]. It is

$$\mathrm{CRLB}(k) = \sqrt{J_{11}^{-1}(k) + J_{22}^{-1}(k)} \tag{61}$$

where J is the information matrix, and it is

$$J(k+1) = \left[F_2^{-1}(k)\right]^T J(k) F_2^{-1}(k) + H_2^T R^{-1} H_2 \tag{62}$$

where $F_2$ and $H_2$ are defined in (28) and (26).

The number of initial scans n is set to 5, 6, 7 and 8, and the standard deviations of measurement noise $\sigma$ in x and y coordinates are set to 5m and 10m, which can convert to the standard deviation of position noise $\sigma_p$ as 7.07m and 14.14m respectively. $\mathbf{Q}_1$ is set as a diagonal matrix with diagonal elements $[1, 1, 0.1, 0.1, 0.01, 0.01]$, and $\mathbf{Q}$ is also a diagonal matrix with diagonal elements $[1, 1, 0.1, 0.01, (0.5\pi/180)^2]$. The test is conducted on 500 Monte Carlo runs. The results are summarized in Table 1.

In Table 1, the RMSE column represents the time averaging on the ensemble RMSE of $p_{err}$. The Noise Redu. column represents the noise reduction percentage, which is computed by

$$\mathrm{NoiseRedu.} = \frac{\sigma_p - \mathrm{RMSE}}{\sigma_p} \times 100(\%) \tag{63}$$

It is seen from Table 1 that all RMSEs are $33\% - 54\%$ lower than the measurement errors. It indicates all the algorithms are performing well. The UKF and the EKF in the proposed nonlinear model generate lower RMSE than the KF in the uncoupled acceleration model, except an EKF case with $n = 5$ and $\sigma_p = 14.14m$. Thus, the proposed nonlinear model outperforms the existing uncoupled acceleration model. It is known that the UKF has better performance than the EKF in nonlinear tracking. This point is further confirmed by the results in Table 1. The UKF shows less RMSE than the EKF in all the cases.

To further analyze the performance difference between the UKF and the EKF on initial conditions, we categorize the initial state accuracy into three levels, namely, excellent, good and poor. We use $\sigma_h$ shown in Figure 1 to represent the initial state accuracy. The excellent level is when $\sigma_h$ is lower than 8 degrees. This level can be reached by the two-stage least squares initiation method when $\sigma_p$ is 7.07m or n is greater than

Table 1: RMSE on Position

| n | Algo. and CRLB | $\sigma_p$=7.07m RMSE (m) | $\sigma_p$=7.07m Noise Redu. (%) | $\sigma_p$=14.14m RMSE (m) | $\sigma_p$=14.14m Noise Redu. (%) |
|---|---|---|---|---|---|
| 5 | KF | 4.16 | 41.2 | 8.17 | 42.2 |
| | EKF | 3.78 | 46.5 | 9.46 | 33.1 |
| | UKF | 3.75 | 47.0 | 7.93 | 43.9 |
| | CRLB | 3.49 | 50.6 | 6.99 | 50.6 |
| 6 | KF | 4.10 | 42.0 | 8.13 | 42.5 |
| | EKF | 3.75 | 47.0 | 7.62 | 46.1 |
| | UKF | 3.74 | 47.1 | 7.48 | 47.1 |
| | CRLB | 3.41 | 51.8 | 6.82 | 51.8 |
| 7 | KF | 4.01 | 43.3 | 8.02 | 43.3 |
| | EKF | 3.67 | 48.1 | 7.44 | 47.4 |
| | UKF | 3.66 | 48.2 | 7.39 | 47.7 |
| | CRLB | 3.34 | 52.8 | 6.67 | 52.8 |
| 8 | KF | 3.95 | 44.1 | 7.94 | 43.8 |
| | EKF | 3.65 | 48.4 | 7.39 | 47.7 |
| | UKF | 3.64 | 48.5 | 7.37 | 47.9 |
| | CRLB | 3.28 | 53.6 | 6.55 | 53.7 |

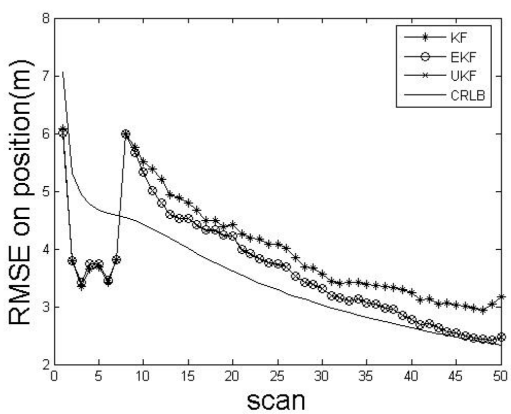


Figure 3: RMSE on position vs. scan when $\sigma_p$=7.07m, n=8 (excellent initial level)

6. The good level is when $\sigma_h$ is around 10 degrees. In our simulation, only the case when $\sigma_p$ and n are 14.14m and 6 respectively, belongs to this level. The poor level is when $\sigma_h$ reaches 15 degrees. It is the worst case in the simulation. In this case, $\sigma_p$ and n are 14.14m and 5 respectively. Figures 3∼5 show the position RMSE vs. time for the three different initial levels. The results are obtained from 500 Monte Carlo runs. The tracking performance for the excellent initial level is presented in Figure 3. It is seen that the UKF and the EKF have

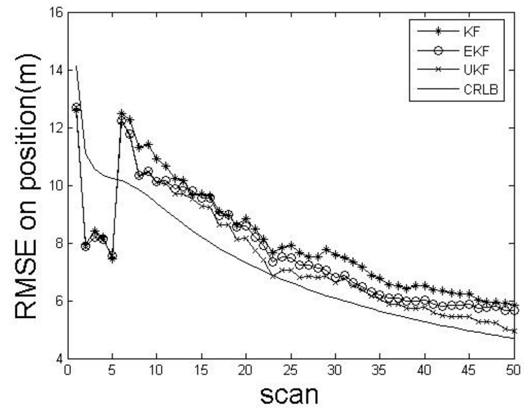


Figure 4: RMSE on position vs. scan when $\sigma_p$=14.14m, n=6 (good initial level)

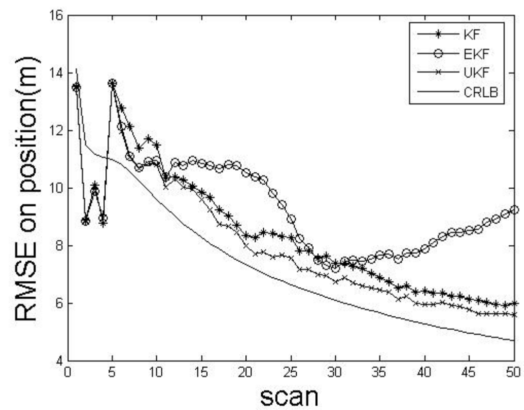


Figure 5: RMSE on position vs. scan when $\sigma_p$=14.14m, n=5 (poor initial level)

comparable performance. Both converge to the CRLB. An obvious gap appears between the linear KF and the two nonlinear filters. Figure 4 shows the RMSE on the good initial level. The UKF obviously outperforms the EKF. Both UKF and EKF are still better than the KF. The result for the poor initiation level is shown in Figure 5. The EKF is diverged. This because the poor performance of the Jacobian approximation when error is large. Although the UKF uses the same inaccurate initial value, the unscented transform is more accurate than a Jacobian approximation, and results in better performance in the UKF. The results confirm that the EKF is more sensitive to the initial condition than the UKF.

From these observations, we can conclude that the proposed nonlinear model performs better than the existing uncoupled acceleration model. Even in the poor initial condition, the nonlinear UKF still outperforms the linear KF. The UKF is less sensitive to the accuracy of the initial state than the EKF. Better nonlinear tracking performance can be obtained by improving the initial state accuracy, and that can be achieved by increasing the number of initial scans.

## 5 Conclusion and future work

This paper proposes a nonlinear constant heading model for tracking a target which moves in a constant acceleration motion. The proposed nonlinear model describes the acceleration motion more accurately than the existing uncoupled acceleration mode, and the error created by the uncoupled linear model is proven. A two-stage least squares method is developed for nonlinear track initiation, and the EKF and the UKF are suggested for state updating. Simulation experiments on the proposed nonlinear model and the existing uncoupled linear model are conducted. The results show that the nonlinear model outperforms the linear model, and the UKF is better than the EKF in terms of track accuracy and sensitivity to the initial condition.

Another method of solving this CA problem is to apply constraint Kalman filter. We shall conduct a research on this method in future.

## References


[1] X. R. Li and V. P. Jilkov, "Survey of maneuvering target tracking. Part I: Dynamic models," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, no. 4, pp. 1333-1364, Oct. 2003.

[2] Y. Bar-Shalom, X. R. Li, and T. Kirubarajan, *Estimation with Applications to Tracking and Navigation: Theory, Algorithms and Software*, New York: Wiley, 2001.

[3] S. S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*, Norwood MA: Artech House, 1999.

[4] R. A. Singer, "Estimating optimal tracking filter performance for manned maneuvering targets," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-6, no. 4, pp. 473-483, Jul. 1970.

[5] S. Dan and C. T. Li, "Kalman filtering with state equality constraints," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 38, no. 1, pp. 128-136, Jan. 2002.

[6] A. Gel, *Applied Optimal Estimation*, Cambridge, MA: MIT Press, 1974.

[7] S. J. Julier and J. K. Uhlmann, "A new extension of the Kalman filter to nonlinear systems," *In Proceedings of AeroSense: The 11th International Symposium on Aerospace/Defence Sensing, Simulation and Controls*, Orlando, FL,1997.

[8] S. J. Julier, J. K. Uhlmann, and H. F. Durrant-Whyte, "A new method for the nonlinear transformation of means and covariances in filters and estimators," *IEEE Transactions on Automic Control*, vol. 45, no. 3 pp. 477-482, Mar. 2000.

[9] S. J. Julier and J. K. Uhlmann, "Unscented filtering and nonlinear estimation," *Proceedings of the IEEE*, vol. 92, no. 3, pp. 401-422, Mar. 2004.

[10] N. J. Gordon, D. J. Salmond, and A. F. M. Smith, "Novel approach to nonlinear/non-gaussian Bayesian state estimation," *Proceedings of IEE*, vol. 140, no. 2, pp. 107-113, Apr. 1993.

[11] B. Ristic, S. Arulampalam, and N. Gordon, *Beyond the Kalman Filter: Particle Filters for Tracking Applications*, Boston London: Artech House, 2004.

[12] C. Jauffret and Y. Bar-Shalom, "Track formation with bearing and frequency measurements in clutter," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 26, no. 6, pp 999-1010, Nov. 1990.

[13] T. Kirubarajan, Y. Bar-Sralom, and D. Lerro, "Bearings-only tracking of maneuvering targets using a batch-recursive estimator," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 37, no. 3, pp. 770-780, Jul. 2001.

[14] T. Brehard and J. P. Le Cadre, "Hierarchical particle filter for bearings-only tracking," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 43, no. 4, pp. 1567-1585, Oct. 2007.

[15] P. Tichavsky, "Posterior Cramer-Rao bounds for discrete-time nonlinear filtering," *IEEE Transactions on Singal Processing*, vol. 46, no. 5, pp. 1386-1396, May. 1998.

[16] T. H. Kerr, "Status of CR-like lower bounds for nonlinear filtering," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 25, no. 5, pp 590-601, Sep. 1989.